

NeMeSiS News 2012

Newsletter of the
ANU-AAMT National Mathematics Summer School

Sugeng rawuh

Welcome to the 14th edition of NeMeSiS News! We hope that you enjoy reading it.

I am always looking for contributions for the newsletter. If you have any feedback, or ideas, or want to write something for the newsletter, or want to tell us what you have been doing since NMSS, please email me. I would love to hear from you.

Merryn Horrocks (editor)

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Vingt ans après

Hi everyone, here I am writing my first article for the NMSS newsletter since becoming director. The title "Vingt ans après" means "Twenty Years Later" and is name of Alexander Dumas' sequel to The Three Musketeers. Twenty years is also how long it has been since I started teaching at NMSS and also how long Terry Gagen served as director. It's also approximately how long Larry Blakers was director. So I thought I might say a bit about things that happen in 20-year cycles. Those that follow popular culture may be aware that 2012 is a significant year in the Mayan Calendar. Indeed, some incorrectly believe that the world will end on Dec 21 of this year. The Mayans believed no such thing, and the maths behind their system is quite nice.

The Mayans used a base-20 system for their calculations. The Mayan year is called a Tun and is 360 days long. A 20 year cycle is called a Ka'tun. A cycle of 20 Ka'tuns is called a Bak'tun. So about every 400 years we come to the end of a Bak'tun. The Mayans believed the world was created on August 11, 3114 BC. So we are about to complete the 12th Bak'tun. Certainly it's a good excuse to dress up, have a party and watch fireworks, but hardly the end of the world. What is the proof that the Mayans thought the world would last longer? Well for one thing they have names for longer cycles! For example, 20 Bak'tuns is called a Piktun; and 20 Piktuns is called a Kalabtun; 20 Kalabtuns is a K'inchiltun; and finally 20 K'inchiltuns makes a

Alautun. So the Mayans have a name for something that is 63 million years long. They'll need to come up with at least two more names to get to the 14 billion years that modern physics predicts for the age of the universe.

So the world will not end in 2012 and neither will NMSS. I have seen two men successfully run NMSS in very different ways, but I have to fill some very big shoes. Staff and students will notice very little change in the academic or social programs, but, behind the scenes, we will start using some of the latest internet technologies. I also hope to invigorate our alumni network. Each year, almost 100 newsletters are returned as people move to new addresses and forget to let us know. With only 64 new alumni each year this means that we are gradually losing touch with our former students and staff. I urge anyone out there who would like to help track down lost alumni and start developing persistent mechanisms for keeping in touch with everyone to contact me with your ideas.

We are the only major summer school activity that has no regular government or corporate sponsorship. Thus, it goes without saying that NMSS is always grateful for any donations however large or small. But I'll say it anyway. Nevertheless, there are many other ways you can help NMSS. Do you have valuable contacts in industry that might help us raise corporate sponsorship? Do you have connections in politics that might raise the status of NMSS in the eyes of the government? Have you yourself had an interesting career that you might wish to share with our students? I love to hear from former students, so please write to me or email me and let's talk about the future.

Leon Poladian

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Why more numbers start with '1'

Look at a list of country population sizes, or the frequency of words in today's newspaper. If you take just the first digit, what do you think you'll see? You may expect to see all the digits from 1 to 9 appear roughly equally, but, remarkably, this is not what happens. In 1881, Simon Newcombe noticed that in books of logarithm tables, the pages at the front (which were used for calculating with 'small' numbers) wore out much more quickly than those at the back. He hypothesised that in any set of data, numbers will tend to begin with '1' more often than any other digit, followed by '2', '3', and so on, with '9' being the least common.

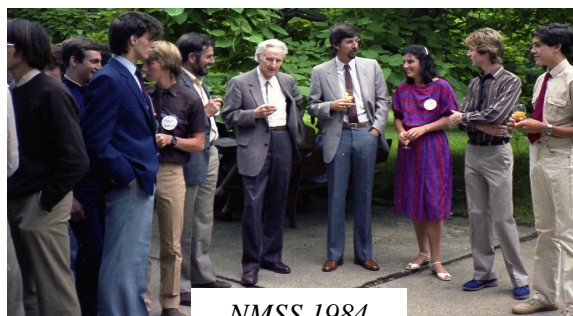
In 1938, Frank Benford re-discovered this fact and collected a large and diverse set of data to verify it. In all cases, the distributions of first digits could be described by the formula,

$$\text{Probability of digit } d = \log_{10}\left(\frac{d+1}{d}\right).$$

This means that '1' occurs about 30% of the time, while '9' is seen less than 5% of the time. Benford's efforts ultimately accorded him naming rights and this is now known as Benford's Law. Although Benford demonstrated its existence, it was far from clear why it occurs with such uncanny regularity.

In 1961, Roger Pinkham made a crucial insight. Suppose you measured the distances travelled by NMSS students to Canberra and look at the first digit. What unit did you use? Should it make a difference? If there was a 'natural' distribution of first digits, you wouldn't expect it to depend on an arbitrary choice like a measurement scale. Pinkham showed that there is only one distribution for which this is true: Benford's Law.

Imagine starting a large number of bank accounts with different amounts of money. Start with the first digits of the bank balances evenly distributed. Now, keep the money in the bank and earn interest until the



money exactly doubles. Every account that previously started with a digit '5' or higher will now start with '1'. More than half of the accounts now have amounts starting with a '1'. Over time, the distribution will gradually converge to Benford's Law.

Is there a secret to Benford's Law? The scale invariance property is its crucial property, but there is an even easier way to understand it. If you take the logarithm of numbers whose distributions follow Benford's Law, you'll notice that now the first digits are equally representative of all numbers from 1 to 9. The uniform distribution was there after all, we just had to look in the right place!

There are many situations when Benford's 'magic' does not apply: for numbers that are systematically generated (like phone numbers), which have a constrained range (like race times for a 200 m sprint), which can be negative (like temperatures in °C) or are designed with a specific distribution (like lottery numbers). For nearly everything else, the data are like moths to Benford's flame ...

There is much more to this story. Terry Tao has a great article on his blog at: <http://terrytao.wordpress.com/2009/07/03/benfords-law-zipfs-law-and-the-pareto-distribution/>

Damjan Vukcevic

Thank You, Terry

On behalf of everyone associated with the NMSS, we would like to thank Professor Terry Gagen, AM, for the many, many years of hard work that he has put into the Summer School. Without him, the Summer School would not be what it is today. He has inspired so many young NMSS mathematicians through his enthusiasm and interest in them. Thank you Terry, and we wish you all the very best for the future.



Quantum Mechanics

One of the most beautiful and mysterious of all physical theories is quantum mechanics. At its core are generalisations of ideas from matrices and vectors ('linear algebra') which describe transformations of space like rotations and stretches. A particle which was a point in classical mechanics now becomes a probability distribution ('wavefunction') with total integral 1, whose integral over any small region gives the probability of the particle being found there. 'Quantisation' transforms physical quantities from classical mechanics like position (x), momentum ($p = mv$) and kinetic energy $\left(E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \right)$ into differential operators (namely, ignoring messy constants, $x \leadsto$ multiplication by x ; $p \leadsto -i\frac{d}{dx}$; $E \leadsto -\frac{d^2}{dx^2}$) which act on these wavefunctions. The allowed quantised values these physical quantities can take are then the so-called 'eigenvalues' of these operators.

For example, for a particle in 1-d trapped in a 'box' between $x = 0$ and $x = 1$, the only allowed solutions of the energy eigenvalue equation $-\frac{d^2 f}{dx^2} = \lambda f(x)$ which are 0 at the boundaries are of the form $f(x) = \sin(\pi m x)$ with $\lambda_m = \pi^2 m^2$; and these λ_m form the discrete levels at which we may measure the particle's energy.

Even the Heisenberg uncertainty principle becomes simply the mathematical statement that the operators $-i\frac{d}{dx}$ (momentum) and multiplication by x (position) don't commute –

$$\text{i.e. } \frac{d}{dx}(xf(x)) \neq x\frac{d}{dx}f(x)!$$



Not the author, but possibly his cat!

Ultimately, the beauty and simplicity of this mathematical formulation itself strongly suggests the correctness of the physics. And it also points the way

to deeper extensions - for example to quantisation of field theories rather than mechanics, such as Feynman's quantisation of electromagnetism and the corresponding theories of the weak and strong nuclear forces.

Richard Stone

"For the Largest Prime"

(to "For the Longest Time" by Billy Joel)

Whoah, Oh, Oh, Oh
For the largest prime
Whoah, Oh, Oh
For the largest ...

I discovered something big tonight:
I don't think that Euclid's proof is right.
What else could I do, I had to show it's untrue
So now I'm searching for the largest prime.

Once I thought that primes went on and on;
Now I know that proposition's wrong.
I've got a feeling, the set of primes it has a ceiling;
I'm sure that I can find the largest prime.

Whoah, Oh, Oh, Oh
For the largest prime
Whoah, Oh, Oh
For the largest ...

Blackboard calculations day and night
In my attic, working out of sight.
I took no chances, didn't want to get wrong answers
'Coz then I'd never find the largest prime.

Every prime I thought was the last
Had me deceived; it soon was surpassed.
My approach gave no effective bound
My method was profound, but it wasn't constructive.

Who knows how much further they'll go on?
Maybe I should use a Pentium.
Poincare and Riemann believed the primes had no supremum,
Yet I'm sure that I can find the largest prime.

I had second thoughts at the start:
Concerns that my proof would fall right apart.
Were the final prime to factorise
I'd be despised, and I'd never get published.

I don't care what consequence it brings:
Implications for sets and groups and rings.
I'm going to find it, explicitly evaluate the
Binary expansion of the largest prime.

Whoah, Oh, Oh, Oh
For the largest prime
Whoah, Oh, Oh
For the largest prime.

David Harvey

NMSS 2012

In January, I was fortunate enough to be a student at the 44th National Maths Summer School, two very memorable weeks of maths, meeting new people and experiencing life at a university college.

At NMSS, I experienced mathematics in a whole new light. I found the maths itself, while very challenging, absolutely fascinating. We covered three courses: Number Theory, the Real Projective Plane and Chaos and Sequences. I loved the fact that the focus was not on getting it right, but on developing our understanding. Whether we got an answer, and if we did, whether we were correct was not so important.

My appreciation of learning for the sake of learning, the sheer joy of attaining new knowledge, grew so much over the course of the program; something I think I can apply not only to mathematics, but also to life in general. I rediscovered a love for the process of understanding, the challenge of those seemingly impossible questions, and how that makes arriving at an answer so much more satisfying.

The lecturers, tutors and NMSS staff in general, exuded such a positive energy and passion for maths, that even after two weeks of very minimal sleep, it was truly contagious. By the end of the first week, the teacher-student relationships had evolved into friendships, and our tutors had become people we could talk to about almost anything.

This was the last cohort that Terry Gagen, Director of the National Mathematics Summer School for 20 years, would take through, as he retired as director at the end of the program. I am sure I speak for the entire 2012 cohort when I say that I feel very privileged to have been able to experience the last NMSS course Terry would ever take; his final lecture, and indeed his final course, was captivating, touching and truly unforgettable.

An aspect of NMSS that I wasn't expecting at all was how incredible the social side of "Maths Camp" was. We were only in Canberra together for two weeks, but when we left, it was as though we'd known each other for a lifetime. We made friends from across

CAN YOU HELP?

As you are aware, NMSS needs all the support that you can give. The school only remains viable because of the donations of past students and their parents. I urge you to make a tax deductible donation if at all possible.

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Thank you for your continued support of NMSS.

Australia, friends who we will keep for a long time yet. Whether it was over meals, maths, table tennis, cards, chess, pool or any number of board games, we all became very close, very quickly. The Experienced Group (or EGs as they were affectionately called) kept us constantly thinking on our feet, with Assassins, Schnapps, Black Magic or an assortment of mind games; the triumphant feeling one gets when one works them out is incomparable!

One of the nicest things about NMSS was the fact that one could go and sit with any group of people and they would be immediately welcomed. The free time we had at the end of each day, the excursions and the middle weekend offered plenty of amazing opportunities to get to know people outside of the lecture theatre.

To spend two weeks in the company of such intelligent, like-minded and downright lovely people was a truly wonderful experience. The program culminated with a formal dinner, a less formal concert and an excellent party; the time to just mingle and enjoy the company of new friends was yet another highlight.

Attending NMSS was one of the most enjoyable and enlightening experiences of my life. The friends I made, the incredible mathematics we were exposed to and the opportunity to be a student in the amazing NMSS program are things for which I am incredibly grateful; things I will never forget.

Kaela Armitage



NMSS 2012